Mathematical Literacy for Tomorrow's Citizens

Implications for Educational Therapy

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Mathematical competence is as important as literacy in today's world. Today, more than ever, we live in a mathematically rich society. Mathematical literacy, also called *numeracy*, empowers individuals to participate fully in their world and contributes to responsible citizenship. Students with learning disabilities are facing increasing challenges in their personal lives as well as in their future careers.

The level of mathematical sophistication expected in the workplace has increased significantly. All occupations require a greater level of mathematical sophistication than ever before, and that trend is increasing. Students with learning disabilities who are not achieving well in mathematics face a growing disadvantage. These students often develop negative views of mathematics, and a resistance that becomes difficult for parents and educators to surmount.

Numeracy Defined

Numeracy refers to a set of skills necessary to think and behave competently in quantitative and spatial realms. The National Council of Teachers of Mathematics (2000) advocates instruction that develops functional mathematical competence. A summary of NCTM recommendations follows.

As a result of an effective mathematics program, an individual can

- Engage in basic computational activities;
- Understand mathematical concepts and make connections among mathematical ideas;
- Represent and analyze mathematical situations using models and symbols;
- Reason mathematically and logically by gathering and analyzing evidence and creating arguments to support or refute hypotheses;
- Communicate precisely about quantities, relationships and unknown values using graphs, charts, mathematical symbols, and words;
- Apply mathematics to everyday life;
- Solve problems that arise in mathematical and other contexts; and
- Appreciate the beauty and power of mathematics.

Functional mathematical literacy requires the ability to access important facts, procedures, concepts, and strategies to solve problems. To do this effectively, students must have been given sufficient practice in each of these areas.

Numeracy Achievement of Students with Learning Disabilities

A number of facts have emerged regarding the numeracy development of students with learning disabilities. Between the ages of 9 and 14, students with learning disabilities make very little progress in computational skills. Warner, Alley, Schumaker, Deshler, & Clark (1980) found that adolescents with learning disabilities improved their numeracy skills the equivalent of only one year during grades 7 through 12. Two studies (Cawley & Miller, 1989; Warner et al., 1980) found that twelfth grade students with learning disabilities, on average, perform on a fifth to sixth grade level in math. Only 12 percent of students with learning disabilities and emotional issues take advanced math classes (algebra, geometry, trigonometry, and calculus) (Wagner & Blackorby, 1996).

Prevalence of Mathematics Disabilities

It is a myth that learning disabilities impair literacy skills more often than numeracy skills. There is evidence of fairly equal prevalence of numeracy and literacy learning problems (Kosc, 1974; Norman & Zigmond, 1980). Light and DeFries (1995) estimate that five to six percent of the school-age population have significant difficulty in mathematics. McLeod and Armstrong (1982) report that two-thirds of students with learning disabilities in grades six to twelve receive some kind of special instruction in mathematics. It is estimated that most individuals who have literacy difficulties have concomitant numeracy deficits (Norman & Zigmond, 1980).

Career Statistics

The twenty professions projected to grow fastest between 2000 and 2010 require knowledge of mathematics (Bureau of Labor Statistics, 2002). Ten of these involve working with computers; the five fastest-growing occupations, such as software engineer and system administrator, involve significant computer knowledge. The computer and data processing services industries are projected to grow the fastest of all industries during this time period. All computer-related occupations require successful completion of math through college-level algebra, and in the more advanced and lucrative computer professions, successful completion of college-level calculus.

The other ten fastest-growing occupations for the same period are in medicine or related fields; these include medical assistant, personal and home care aide, physician assistant, fitness trainer, and aerobics instructor. Many of our students will wish to pursue careers requiring post-secondary education. Persistent difficulties in mathematics can severely limit their career choices. In light of this, students must be sufficiently prepared to pass the

high school and college math courses that will enable them to pursue a wide range of options.

Some Factors Contributing to Poor Numeracy Achievement

A number of factors contribute to the poor numeracy achievement of students with learning disabilities. These include: 1) insufficient research in numeracy issues for students with learning disabilities, 2) students' learning difficulties, 3) negative reactions to mathematics, 4) instructional inadequacies, and 5) limited diagnostic information.

Limited Research in Numeracy Issues

Approximately five to six percent of students suffer from persistent difficulty learning mathematics, yet there are very few studies delineating the factors contributing to poor numeracy achievement and best teaching practices for these students. Research has focused largely on elementary school children and basic number, counting, and arithmetic skills. Less is known about impairments in development in other domains of mathematics. A review of research studies reported between 1975 and 1997 identified only 54 studies of the math performance of students with learning disabilities (Miller, Butler & Lee, 1998). This is significantly fewer than the number of studies conducted on literacy development.

Learning Difficulties

The research in poor growth of numeracy skills suggests that learning difficulties may be related to deficits in executive function processes, semantic memory representation and retrieval, language functions, and visual-spatial skills.

Executive function processes play a crucial role in the development of numeric literacy. Many students with learning disabilities have trouble with problem representation, which involves the conversion of verbal and numeric information into a mental representation of structural relationships. These students have trouble identifying and instantiating the mathematical schema necessary for translating word problems into equations. They also can have difficulty evaluating their ability to solve math problems, organizing math information, identifying and choosing strategies, monitoring their problem-solving techniques, and evaluating the accuracy of solutions.

Active working memory is another executive control process whose deficit significantly impairs numeric competence. It plays a major role in the acquisition of conceptual, procedural, and strategic knowledge. Both verbal and visual-spatial components of the working memory system function in math learning. Working memory exerts a wide influence in the acquisition of numeric competence, including basic fact memorization, development of counting principles and strategies, and attainment of computation procedures.

Many students experiencing learning problems in both numeracy and literacy have more difficulties in mathematics than students with only a mathematics disability. Students experiencing difficulties in both areas may have deficits representing in semantic memory the phonological features of spoken numbers and the semantic aspects of the number system, which lead to basic fact-learning and -retrieval difficulties. They also may have trouble internalizing language to regulate behavior. This leads to difficulty generating concept imagery, developing problem-solving strategies, and guiding problem-solving experiences.

Individuals with language difficulties may have trouble acquiring mathematical facility. They may have problems learning vocabulary and syntax, sequencing numeric and textual elements, becoming skilled in executing algorithms and procedures; processing multiple word meanings, and decoding and comprehending word problems. They may not understand that equations represent relationships rather than directions for completing actions.

Students with mild to profound difficulty processing and representing visual and spatial information also frequently have trouble in math. Problematic areas include forming visual imagery; creating spatial representations of quantitative data; sequencing according to number, length, area or volume; understanding the number line; developing a sense of the number system; interpreting pictorial information; integrating directional aspects of written algorithms; and attending to operation signs.

Negative Feelings About Mathematics

Numeracy problems tend to be more tolerated than literacy problems in our society. Many adults harbor negative feelings about math. Burns (1998) estimates that 66 percent of all U.S. adults have *strong* negative feelings about math that are consciously or unconsciously communicated to young people.

Negative attitudes about math among teachers or parents can contribute to tolerance for poor math achievement. These attitudes may also hinder the search for appropriate diagnostic and remedial services. The National Council of Teachers of Mathematics (2000) points out that math anxiety is a problem for many students and encourages teachers to help develop positive attitudes. Unfortunately, some classroom practices inculcate boredom, feelings of incompetence, and fear, which can lead to failure. This can cause lowered academic self-concept, expectations of poor future achievement, and maladaptive attributions that ascribe failure to personal deficiencies, or success to luck. Students who attribute success to effort persist longer and make greater academic progress. Identifying and addressing maladaptive attributions are important to improve numeracy achievement.

Problems With Instruction

Children enter school with mathematical understandings developed through meaningful interactions with

their environments. Their "street" math knowledge is the foundation of formal (school) mathematics growth. For many students with learning disabilities there is a weak transfer of knowledge and skills from real-life experiences to formal school contexts as well as from school to real-life situations.

Lyon (1996) has shown that learning numeracy concepts, more than any other area, is closely tied to the teacher's knowledge of mathematical concepts and appropriate teaching methods. Many teachers are uncomfortable teaching mathematics and devote significantly less instructional time than is necessary for student success. Even teachers who know the subject may have inadequate pedagogical skills to instruct students with special math needs.

According to the Third International Math and Science Study, (International Association for the Evaluation of Educational Achievement, 1996) the numeracy achievement levels of U.S. students are below the international average. A comparative analysis of the teaching practices between the U.S. and six nations that outperform the U.S. (Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland) revealed that U.S. students spend significantly more seatwork time practicing routine computations and considerably less time inventing new solutions, proofs, and procedures. (National Center for Education Statistics, 2003) Teachers in these countries emphasize instruction in both procedural and conceptual aspects of math, while U.S. teachers tend to reduce conceptual problems into exercises in following directions to find the correct answer. Further, U.S. teachers seldom provide authentic, real-life problem-solving experiences; they restrict problem-solving instruction to word problems.

Although textbooks have improved, many are still inadequate in developing concepts, connecting mathematical ideas, practice and review, emphasis on real-world applications, and instruction in the use of strategy.

Formal Diagnostic Procedures Underemphasize Math

Children are rarely referred for evaluation solely because of difficulties in learning mathematics. Students with numeracy problems who do not also exhibit literacy difficulties often are not referred for testing or special services. This may be a sign of tolerance for numeracy difficulties or inadequate awareness of the numeracy demands in later life.

Current diagnostic evaluations provide valuable information about a student's mathematical skills that can be used for treatment planning. However, they have shortcomings because they mirror the focus on computation and word problem-solving prevalent in this country. They provide limited data regarding a student's conceptual development, strategy use, procedural "bugs," or use of manipulatives and other resources. Inadequate information about mathematical competence impacts the utility of test results.

Diagnostic instruments such as the Woodcock Johnson or Key Math provide only general guidelines for treatment planning. For effective treatment planning, formal diagnostic test information must be supplemented with informal assessment of the student's strengths and weaknesses in conceptual understanding, procedural use, and problem solving (including strategy use) using behavioral observation, error analysis, and clinical interviews. Furthermore, students' affective dispositions toward math and their mathematical competence have to be determined.

Role of the Educational Therapist

Educational therapists should examine their beliefs to see if they harbor negative feelings toward mathematics. They should also consider whether they accord numeracy the same status as literacy in therapy. They are in a position to realize the importance of math education in an increasingly technological world. Consequently, educational therapists ought to devote sufficient resources to the development of skills needed for independent functioning in employment, finance, health, interpersonal relations, and educational environments. Instruction for students who have numeracy difficulties must be a twenty-first century goal.

Treatment plans should be based upon the integration of formal and informal diagnostic information. Formal diagnostic data should be augmented by a thorough assessment of three types of mathematical knowledge: conceptual (including "street" math skills), procedural, and strategic. As students with numeracy difficulties probably lack sufficient foundational knowledge, significant remedial attention must be given to the development of core conceptual, procedural, and strategic knowledge.

Students' attitudes and beliefs regarding their capabilities in math must be evaluated because these attitudes affect their efforts and achievement of goals. The most effective instruction is based upon a solid understanding students' attitudes.

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